

# Quadratics: see the roots

Complete the square, interpret a minimum, and use the discriminant. Show your reasoning; no calculator is needed. Worked answers start on page 3.

## QUESTIONS / 1 OF 2

### 1 Read a minimum

Write  $x^2 - 8x + 3$  in the form  $(x - a)^2 + b$ . State its minimum value and the value of  $x$  at which it occurs.

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### 2 Keep the coefficient outside

Write  $2x^2 + 12x + 7$  in the form  $a(x + b)^2 + c$ . Hence state the turning point of the graph  $y = 2x^2 + 12x + 7$ .

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### 3 Three values, three outcomes

For  $x^2 - 6x + k = 0$ , complete the table. Then find any real roots when  $k = 8$  and when  $k = 9$ .

| $k$ | Discriminant $\Delta$ | Number of real roots |
|-----|-----------------------|----------------------|
| 8   |                       |                      |
| 9   |                       |                      |
| 10  |                       |                      |
|     |                       |                      |
|     |                       |                      |

# Explain the boundary

Use complete-square form or the discriminant where appropriate. State whether the boundary values are included.

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## QUESTIONS / 2 OF 2

### 4 The core question

Find all real values of  $k$  for which  $x^2 - 6x + k = 0$  has no real roots. Give a complete-square argument and check it using the discriminant. Explain what happens at  $k = 9$ .

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### 5 When a line touches a curve

The line  $y = 2x + c$  meets the curve  $y = x^2 - 4x + 5$ . Find  $c$  when the line is tangent, and find the point of contact. State the values of  $c$  giving two distinct intersections.

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### 6 A parameter in the middle

Find all real values of  $p$  for which  $x^2 + px + 9 > 0$  for every real  $x$ . Explain why the endpoints of your interval are excluded.

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# Check the method

Compare the reasoning as well as the final result. An equivalent correct method is just as useful.

## WORKED ANSWERS / 1 OF 2

### 1 Read a minimum

1. Half of  $-8$  is  $-4$ ; its square is  $16$ .
2.  $x^2 - 8x + 3 = (x - 4)^2 - 16 + 3 = (x - 4)^2 - 13$ .
3. The squared term is non-negative and is zero at  $x = 4$ . The minimum is  **$-13$** , at  $x = 4$ .

### 2 Keep the coefficient outside

1. Factor the quadratic and linear terms:  $2(x^2 + 6x) + 7$ .
2. Complete the square inside the brackets:  $2((x + 3)^2 - 9) + 7$ .
3. Simplify to  $2(x + 3)^2 - 11$ . The minimum occurs at  $x = -3$ , so the turning point is  $(-3, -11)$ .

The outer 2 multiplies the correction  $-9$  too.

### 3 Three values, three outcomes

1. Use  $\Delta = b^2 - 4ac = (-6)^2 - 4(1)(k) = 36 - 4k$ .
2. Positive, zero and negative discriminants give two distinct, one repeated and no real roots respectively.
3. For  $k = 8$ , factor  $(x - 2)(x - 4) = 0$ . For  $k = 9$ , use  $(x - 3)^2 = 0$ .

| $k$ | Discriminant $\Delta$ | Number of real roots | Roots         |
|-----|-----------------------|----------------------|---------------|
| 8   | 4                     | Two distinct         | 2 and 4       |
| 9   | 0                     | One repeated         | 3 (repeated)  |
| 10  | -4                    | None                 | No real roots |

# Make the conditions precise

Strict inequalities exclude a curve that only touches the x-axis. Check the equality case explicitly.

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## WORKED ANSWERS / 2 OF 2

### 4 The core question

1.  $x^2 - 6x + k = (x - 3)^2 + k - 9$ ; its minimum is  $k - 9$  at  $x = 3$ .
2. The upward-opening curve stays above the axis exactly when  $k - 9 > 0$ . Therefore  $k > 9$ .
3. Check:  $\Delta = 36 - 4k < 0$  also gives  $k > 9$ . At  $k = 9$ ,  $(x - 3)^2 = 0$  has the repeated real root  $x = 3$ ; exclude it.

### 5 When a line touches a curve

1. Equate the two expressions for  $y$ :  $x^2 - 6x + 5 - c = 0$ .
2. Tangency requires a repeated root:  $\Delta = 36 - 4(5 - c) = 16 + 4c = 0$ . Thus  $c = -4$ .
3. Then  $(x - 3)^2 = 0$ , so  $x = 3$  and  $y = 2(3) - 4 = 2$ . The contact point is  $(3, 2)$ .
4. Two distinct intersections require  $16 + 4c > 0$ , so  $c > -4$ .

### 6 A parameter in the middle

1. Complete the square:  $x^2 + px + 9 = (x + p/2)^2 + 9 - p^2/4$ .
2. The minimum must be strictly positive:  $9 - p^2/4 > 0$ . Hence  $p^2 < 36$ , giving  $-6 < p < 6$ .
3. At  $p = 6$  or  $p = -6$ , the expression is  $(x + 3)^2$  or  $(x - 3)^2$ . It equals zero at one real  $x$ , so the strict condition fails.

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